Developing and evaluating animations for teaching quantum mechanics concepts

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HEA UK Physical Sciences seminar, University of Sheffield, 8 February 2011

"I understand the material, I just can't do any of the problems."

Second year physics student in tutorial

"You cannot become a marathon runner by watching marathons on TV"

(Eric Mazur, Physics World, February 2009)

"If a student understands the material, they can take it and apply it to something new and completely different."

Prof Ian Bonnell, St Andrews

Outline

- Conceptual understanding
- Misconceptions in Quantum Mechanics
- Usefulness (and limitations) of animations
- Repositories and evaluation of multimedia materials
- The St Andrews QM animations project
- Future plans

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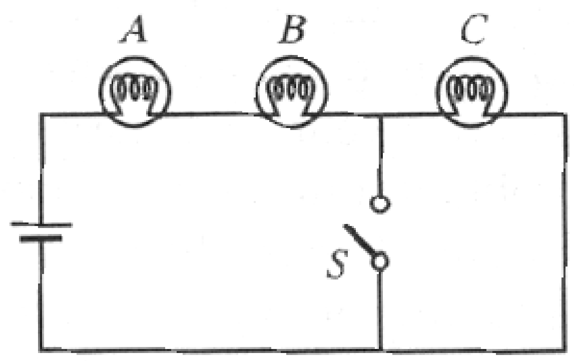
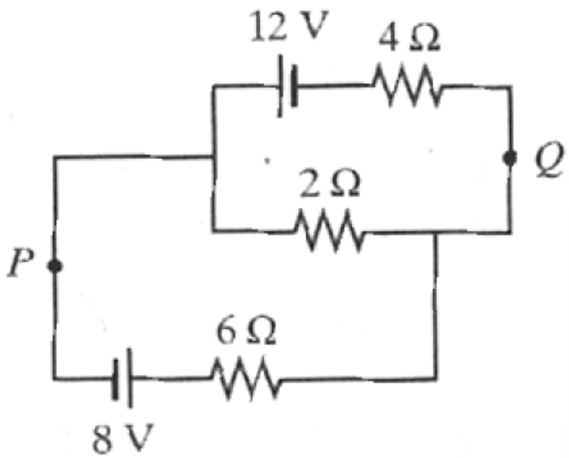
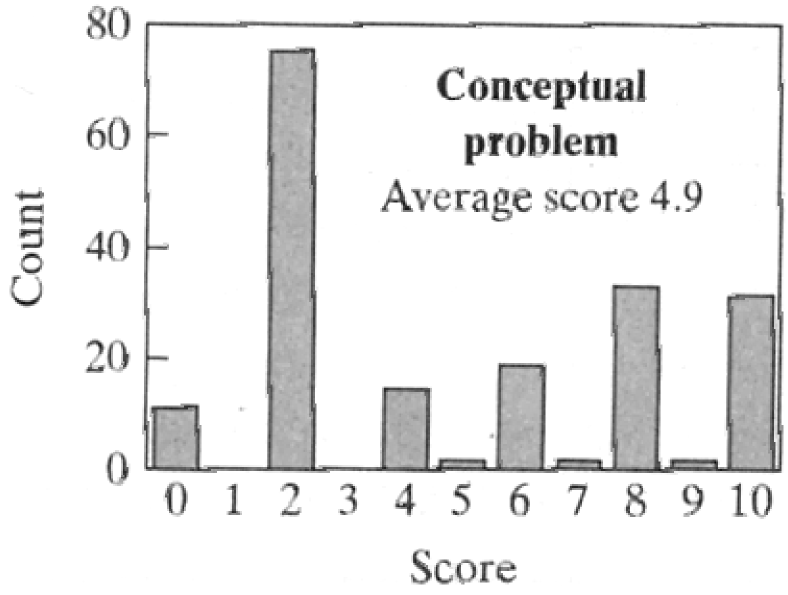
When the switch S is closed, which of the following increase, decrease, or stay the same? intensities of bulbs A and B, intensity of bulb C, current drawn from the battery, voltage drop across bulb A, total power dissipated

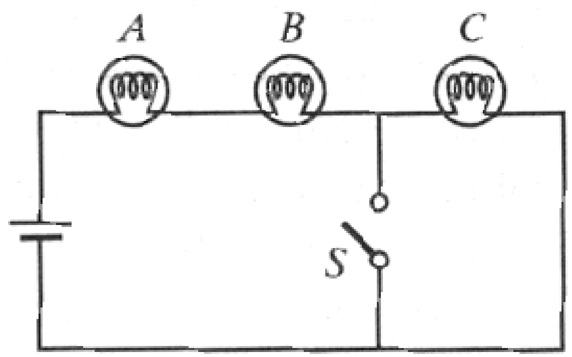


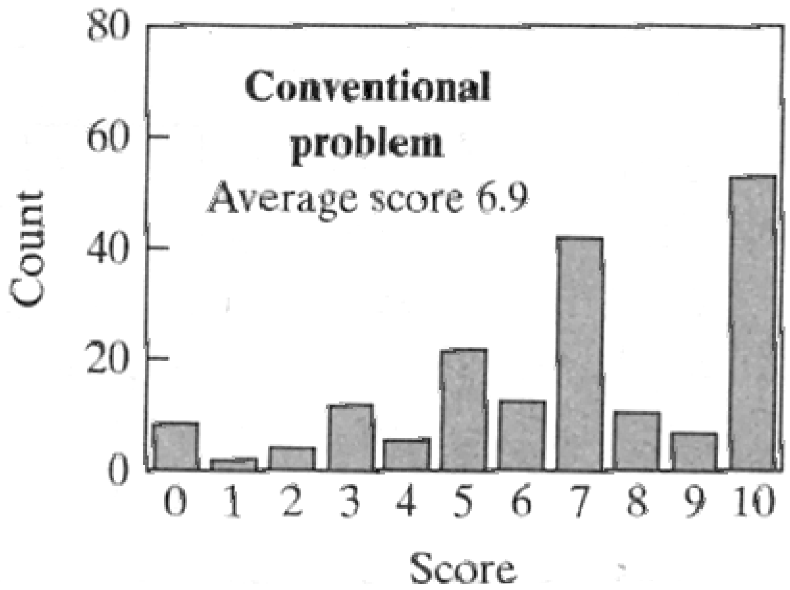


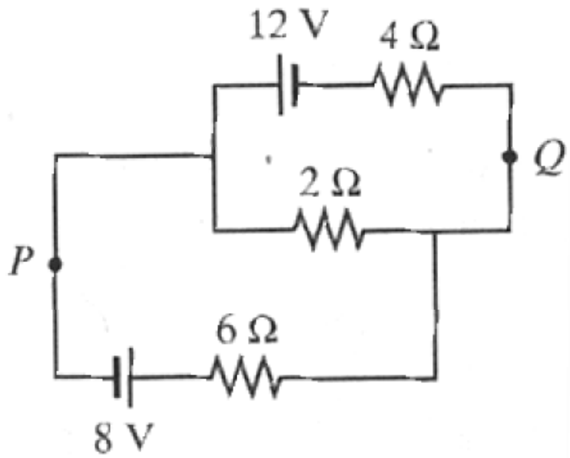
Calculate the current in the 2 Ω resistor and the potential difference between points P and Q.

[E Mazur, Peer Instruction, 1997]









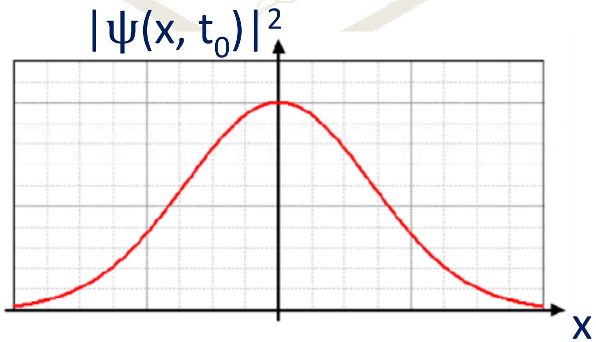
[E Mazur, Peer Instruction, 1997]

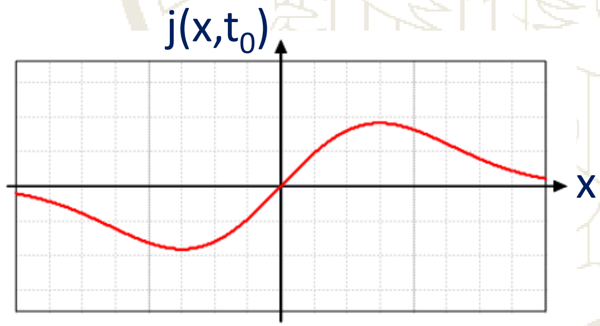
An example: 1D probability current

1) Calculate the probability current for the wave function $\psi(x) = Ae^{ikx} + Be^{-ikx}$

$$j = \frac{\hbar}{2im}\left(\psi^* \frac{\partial \psi}{\partial x} - \frac{\partial \psi^*}{\partial x}\psi\right)$$

2) Given graphs of the probability density and the probability current at time t_0, sketch qualitatively the probability density at t_0+dt.





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Conceptual understanding

- It is possible for students to do well on conventional problems by memorizing algorithms without understanding of the underlying physics.
- Conceptual knowledge can improve a student's ability to perform calculations. It is decisive for those problems requiring a transfer of knowledge to new contexts.

[Carr and McKagan, Am J Phy, 2009, E Mazur, Peer Instruction, 1997, Thacker et al, Am J Phys 1994]

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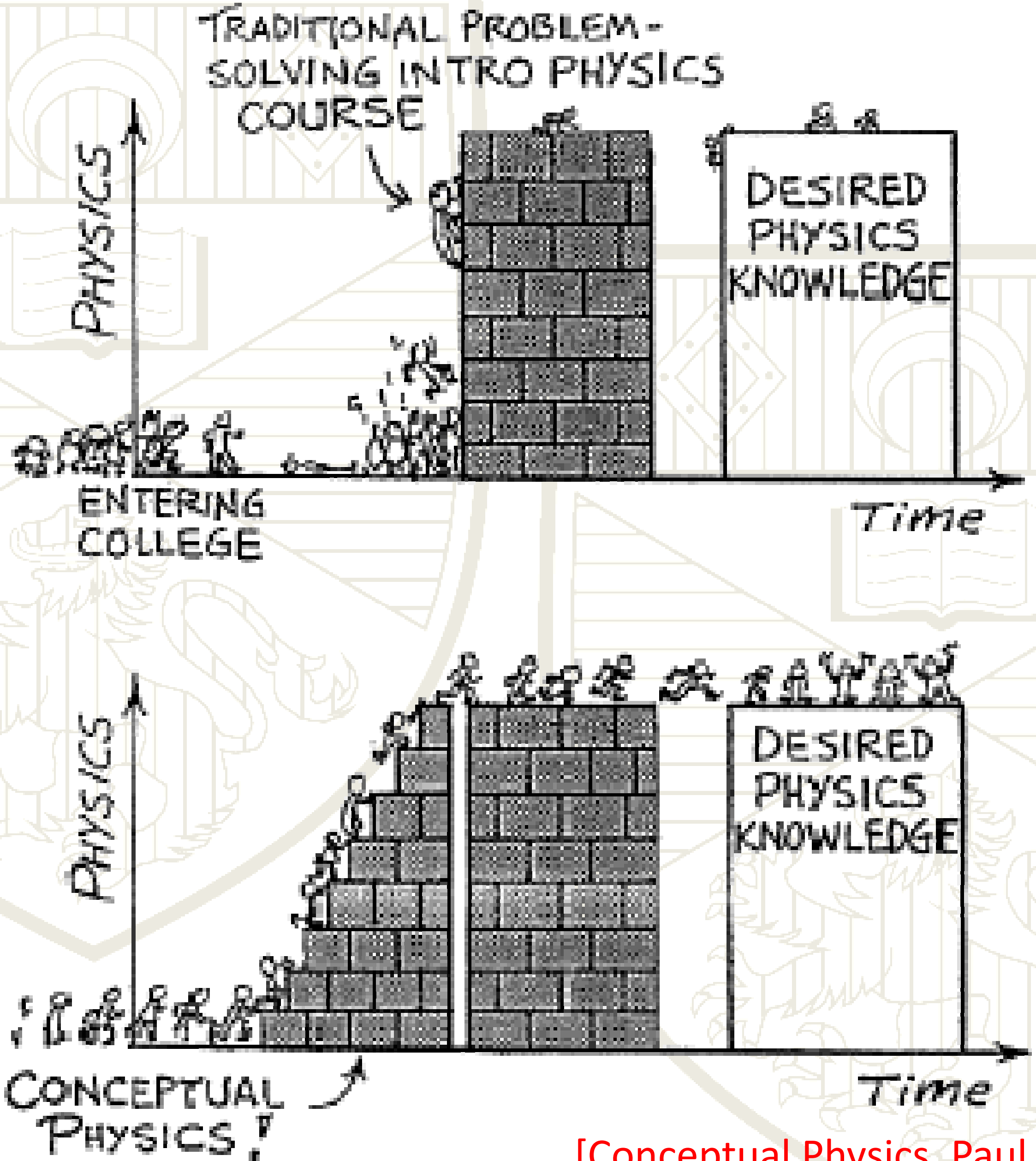

[Conceptual Physics, Paul G. Hewitt, Instructor's Manual]

Student perceptions

"I think I can safely say that nobody understands quantum mechanics"

Feynman, 1967, *The Character of Physical Law*, p.129

- Phenomena often not directly observable
- Introductory topics (eigenenergy problem) often viewed as abstract and far-removed from reality
- Studies of misconceptions and student difficulties have found similarities across diverse populations of students

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Misconceptions in quantum mechanics

Underlying reasons for misconceptions

- False analogies with classical systems Wittmann et al., *Eur. J. Phys*., **26** (6), 939-950, 2005
- Overgeneralization of concepts into an area where they are not directly applicable Singh et al., *Am. J. Phys.*, **76** (3), 277-287, 2008
- Confusion between related concepts Singh et al., *Am. J. Phys.*, **69** (8), 885-895, 2001
- Difficulties with probabilistic interpretations Domert et al., *Eur. J. Phys*., **26**, 47-59, 2005, Bao et al., *Am. J. Phys*., **70** (3), 210-217, 2002

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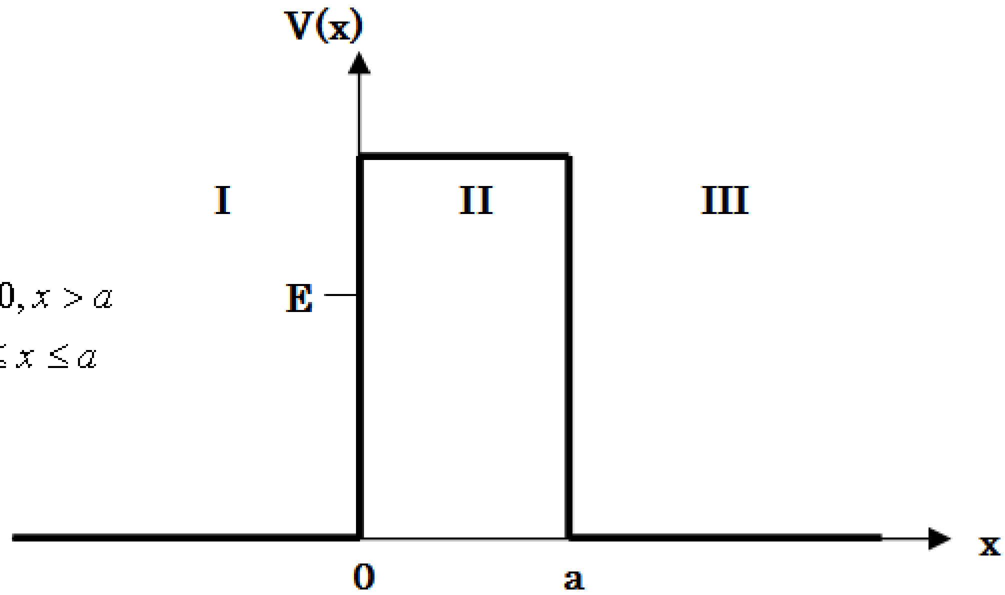
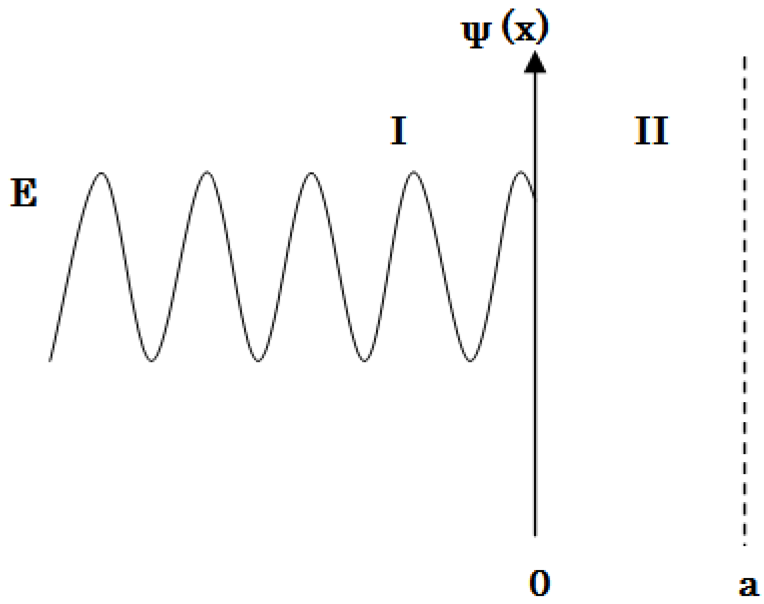
Quantum tunneling

Fill in the blanks in the plot of ψ(x) vs x below, sketching the x-axis and the form of the electron's wave function in regions II and III.

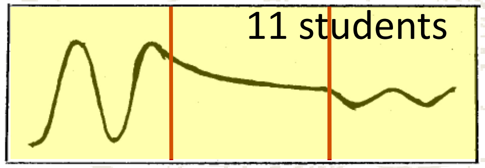
$E < V_0$

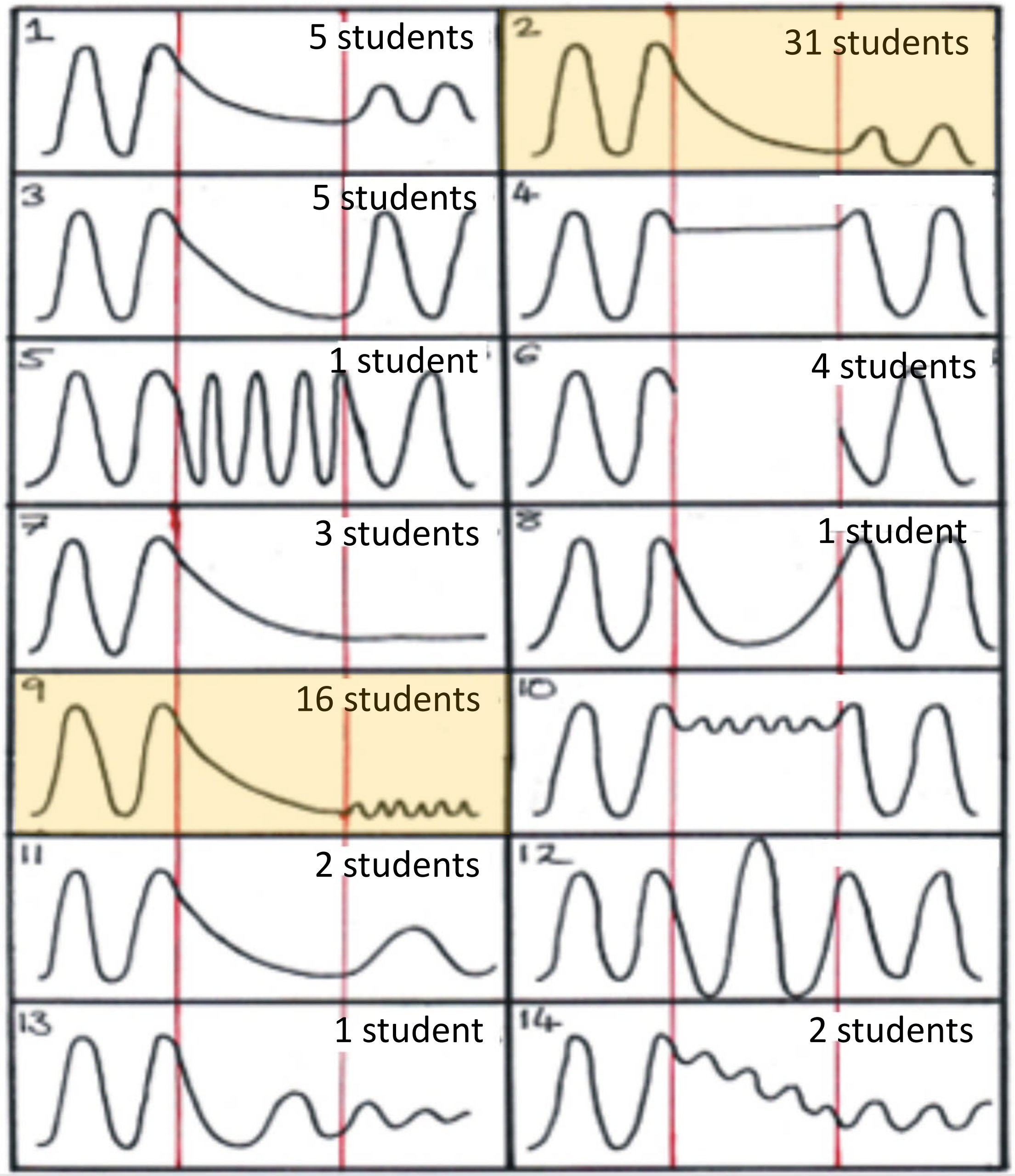
$$V(x) = \begin{cases} 0 & x < 0, x > a \\ V_0 & 0 \le x \le a \end{cases}$$





Results for $3^{rd}/4^{th}$ year: Quantum tunneling

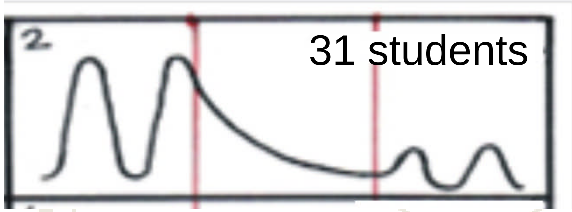




[Robertson and Kohnle, GIREP proceedings, 2010]

see also McKagan et al, *Phys. Rev. ST Phys. Educ. Res.*, **4**, 020103-1-18, 2008

Results: Quantum tunneling

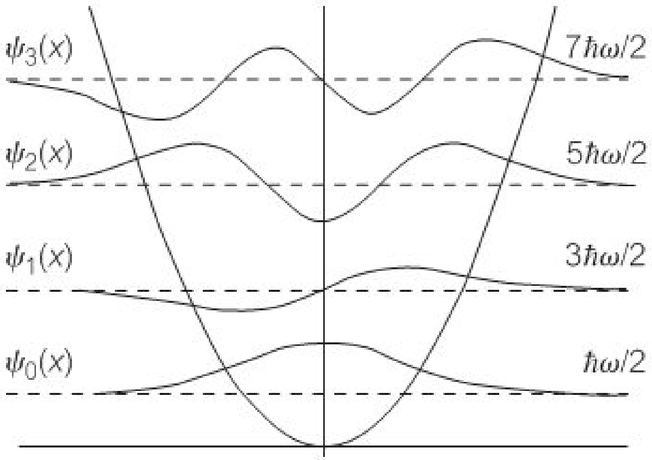


"It continues on the other side with decreased energy"

17% of comments (9% of all students) state that the total energy of an electron decreases when tunneling through the barrier

- Confusion of relations amplitude / probability and wavelength / energy
- False analogy with classical systems (for a classical wave, the energy *does* depend on the amplitude) such as bullets going through a wall.

Danger of plots overlaying V and ψ?



Animations versus demonstrations

- Animations can constrain students' focus on the aspects experts believe are most important. (Finkelstein et al., Phys Rev ST Phys Ed Res, 1, 010103-1-18, 2005)

- Animations can show what is not visible to the eye.

- Interactivity is vital in making animations an effective learning too. Demonstrations have limited educational benefit compared with animations. (Crouch et al., Am J Phys, 72, 835, 2004)

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Key features of educationally effective animations

- Design that encourages and guides the discovery process (interactivity, scaffolding in terms of parameters that can be changed)
(Wieman et al., *Am J Phys* 76, 393-399, 2008)

- Content avoids cognitive overload, peripheral information which can obscure understanding. Animation should not look boring or intimidating.
(Finkelstein et al., *Phys Rev ST Phys Ed Res*, 1, 010103-1-18, 2005)

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Key features of educationally effective animations

- Animation enables to directly link multiple representations (physical motion, vectors, graphs, mathematics). This facilitates making connections and enhances understanding
- Effect of students' perceived prior knowledge: The more students believe already know about the topic, the less they engage with the animation. (Adams, et al. *J. Interactive Learning Research,* **19** (3), 397-419, 2008)

Testing with students is essential!

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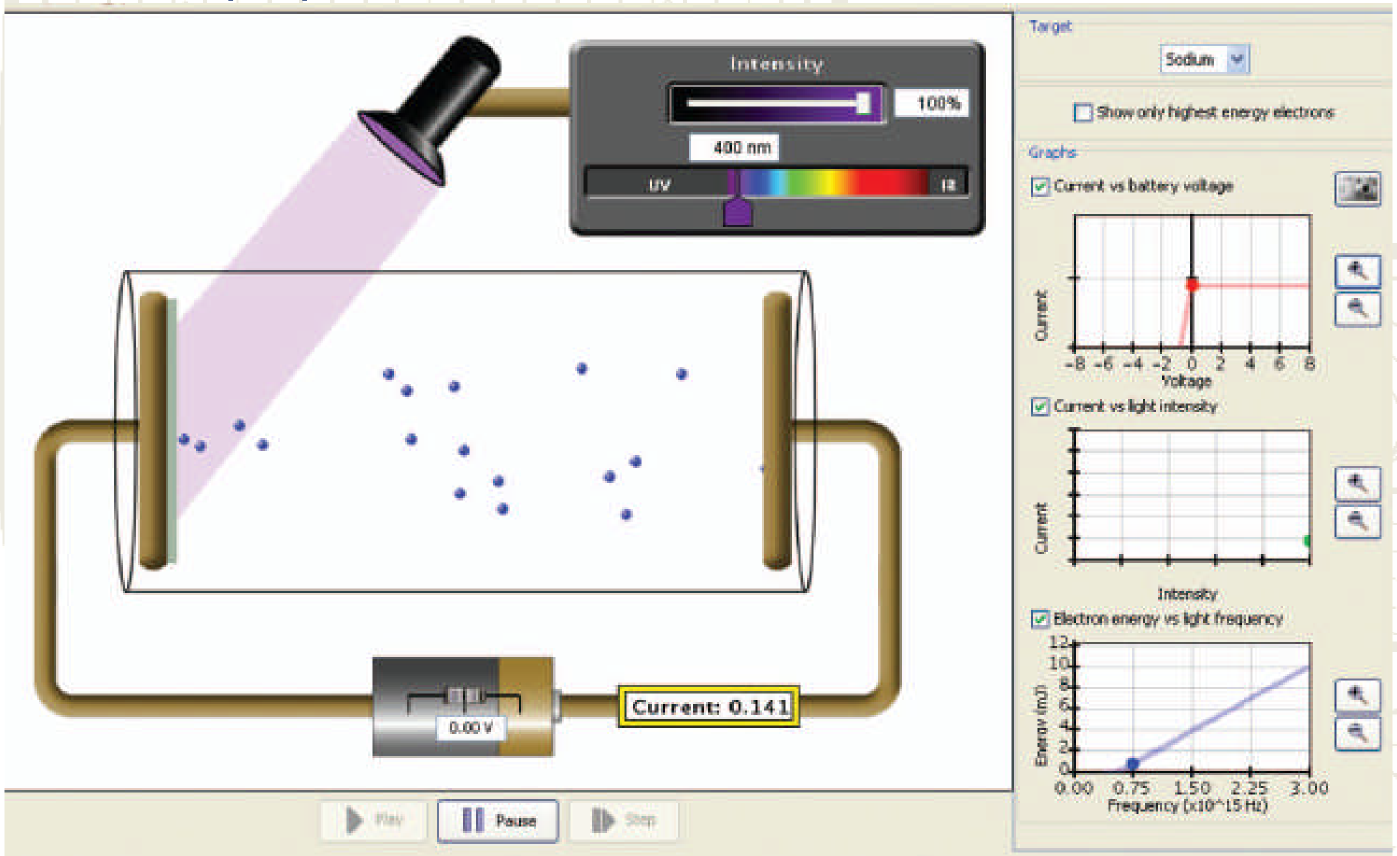
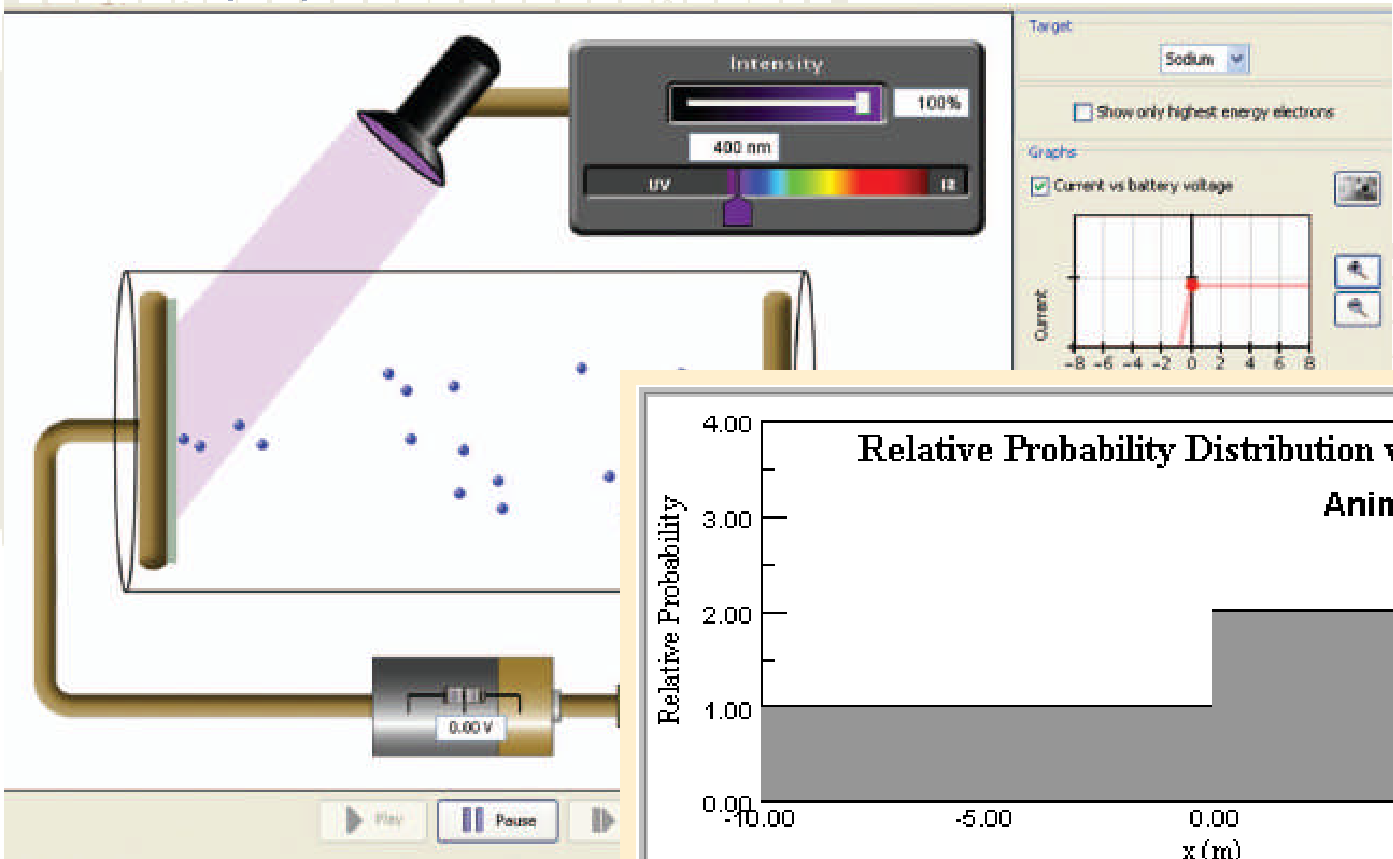
PhET: http://phet.colorado.edu

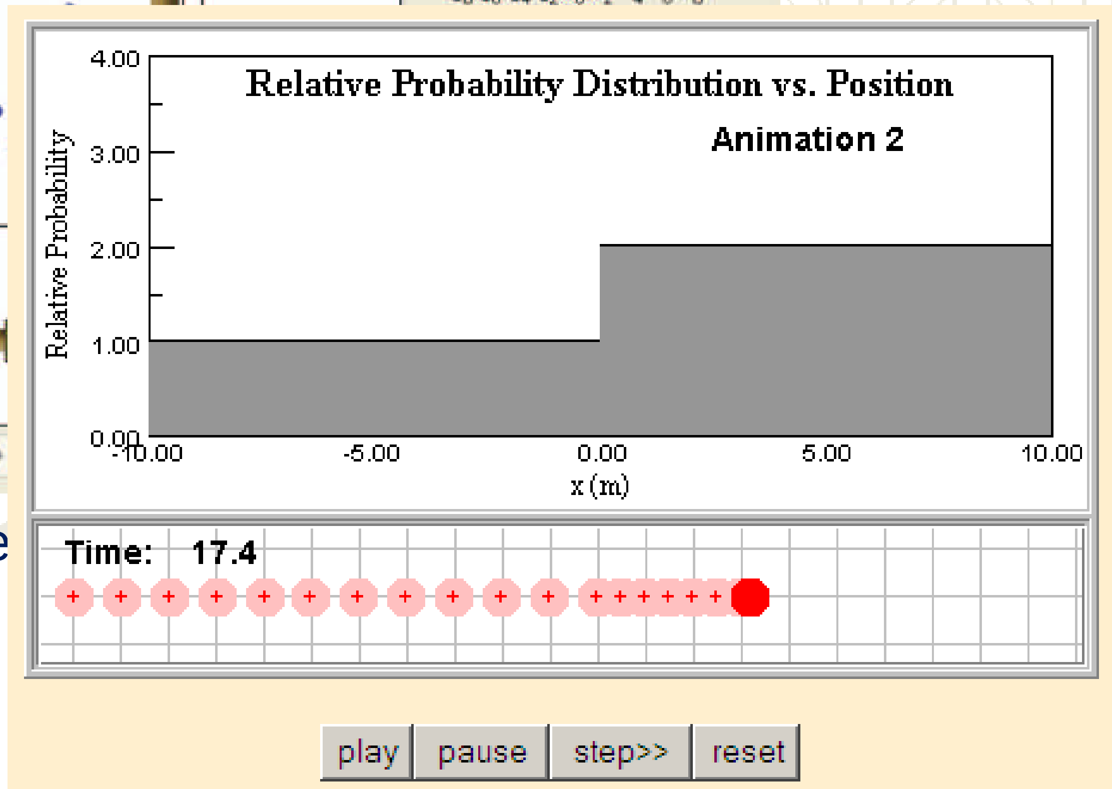


The photoelectric effect

PhET: http://phet.colorado.edu



The photoe



Classical probability distributions

Physlets: http://webphysics.davidson.edu/physlet_resources/

Evaluation of multimedia resources

MPTL

Multimedia in Physics Teaching and Learning

Evaluation criteria:

- Quality of content
- Potential effectiveness for learning
- Ease of use

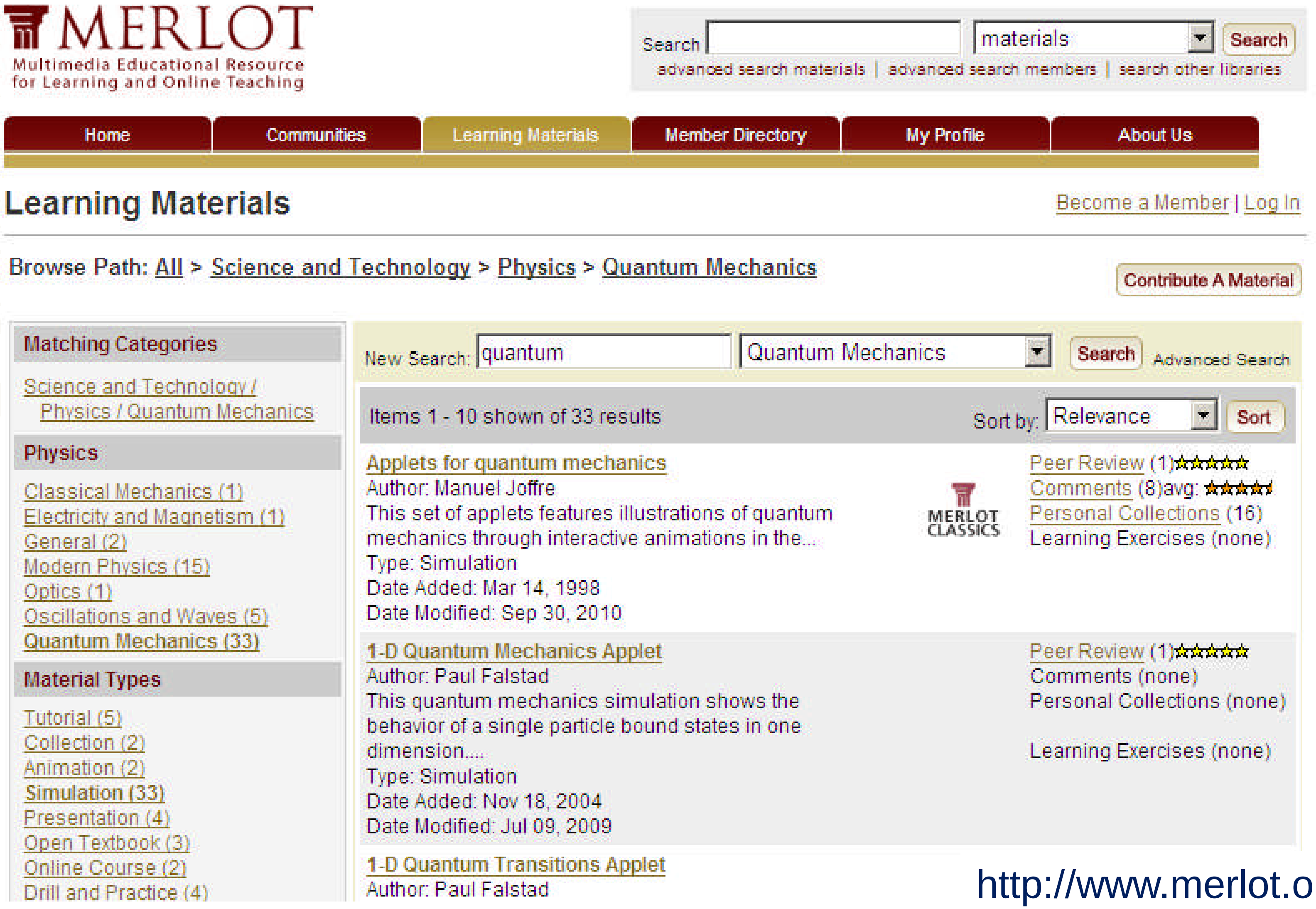
MPTL > evaluations of MM

Reports and recommendations on available multimedia material

Workshop	Title
14th Udine 2009	Optics and Waves
12th Wroclaw 2007	Solid State, Nuclear and Particle Physics
11th Szeged 2006	Electricity and Magnetism
10th Berlin 2005	Statistical and Thermal Physics
9th Graz 2004	Mechanics
8th Prague 2003	Optics
7th Parma 2002	Quantum mechanics Report Recommendations

http://www.mptl.eu/evaluations.htm

Evaluation of multimedia resources



http://www.merlot.org

HEA PSSC Development Project 2009/10: Enhancement of Student Conceptual Understanding of Quantum Mechanics through the Development of Animated Visualisations ...

Antje Kohnle, Margaret Douglass, Tom Edwards,
Aly Gillies, Chris Hooley, Bruce Sinclair

School of Physics and Astronomy
University of St Andrews, UK

(Kohnle et al., *Eur J Phys,* 31, 1441-1455, 2010)

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Project aims

- Develop animations to help students build visual representations of quantum mechanics concepts
- Target known misconceptions and student difficulties from the education research literature and our own work
- Make animations and instructor resources freely available

27 Animations developed so far

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Overview of the animations

- Complementary to existing animations (Physlets, PHET, ...)
- Developed in Flash, graphics imported from Mathematica → only require Flash Player to run
- Small file size, typ. 80 kB (1D), 2 MB (3D)

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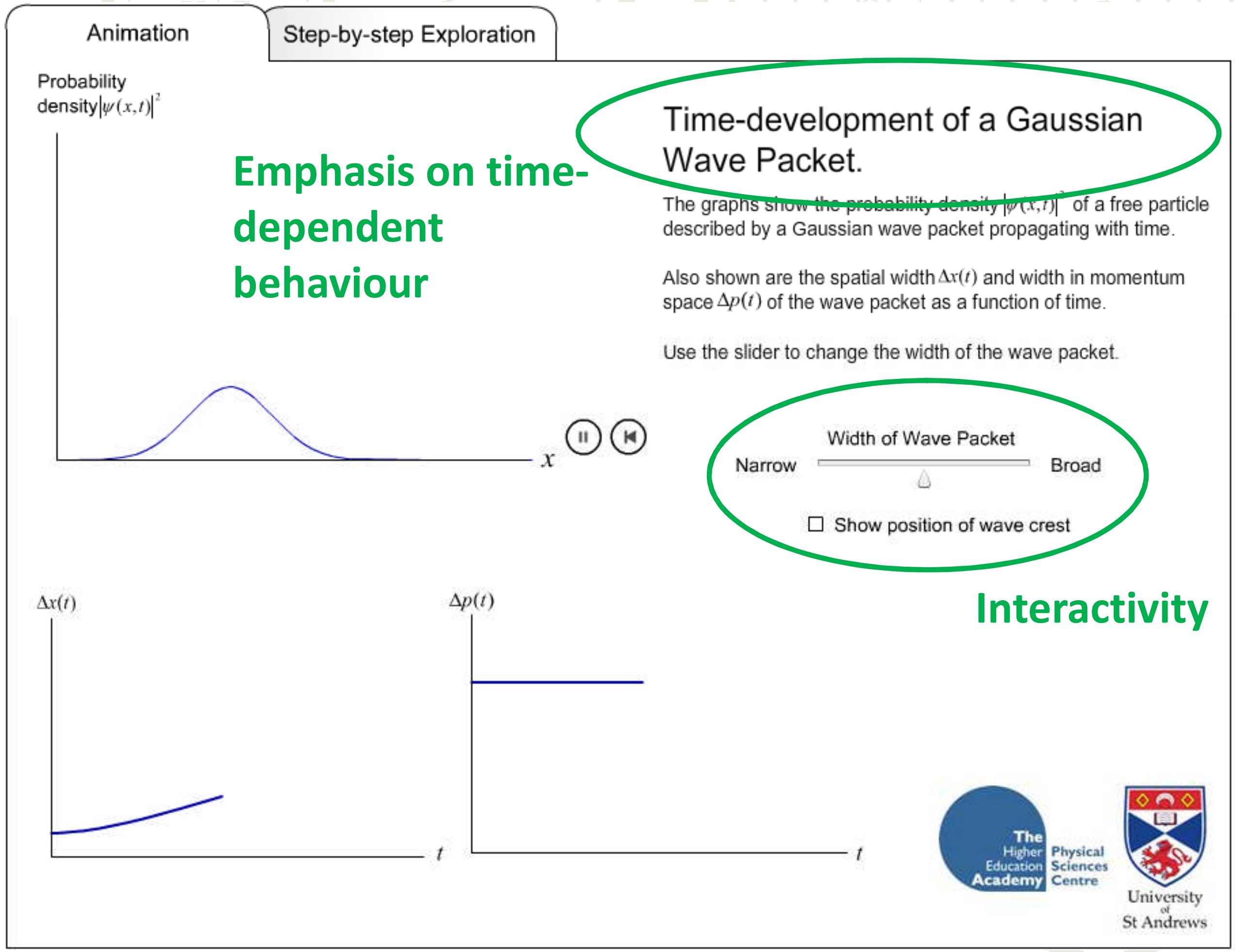
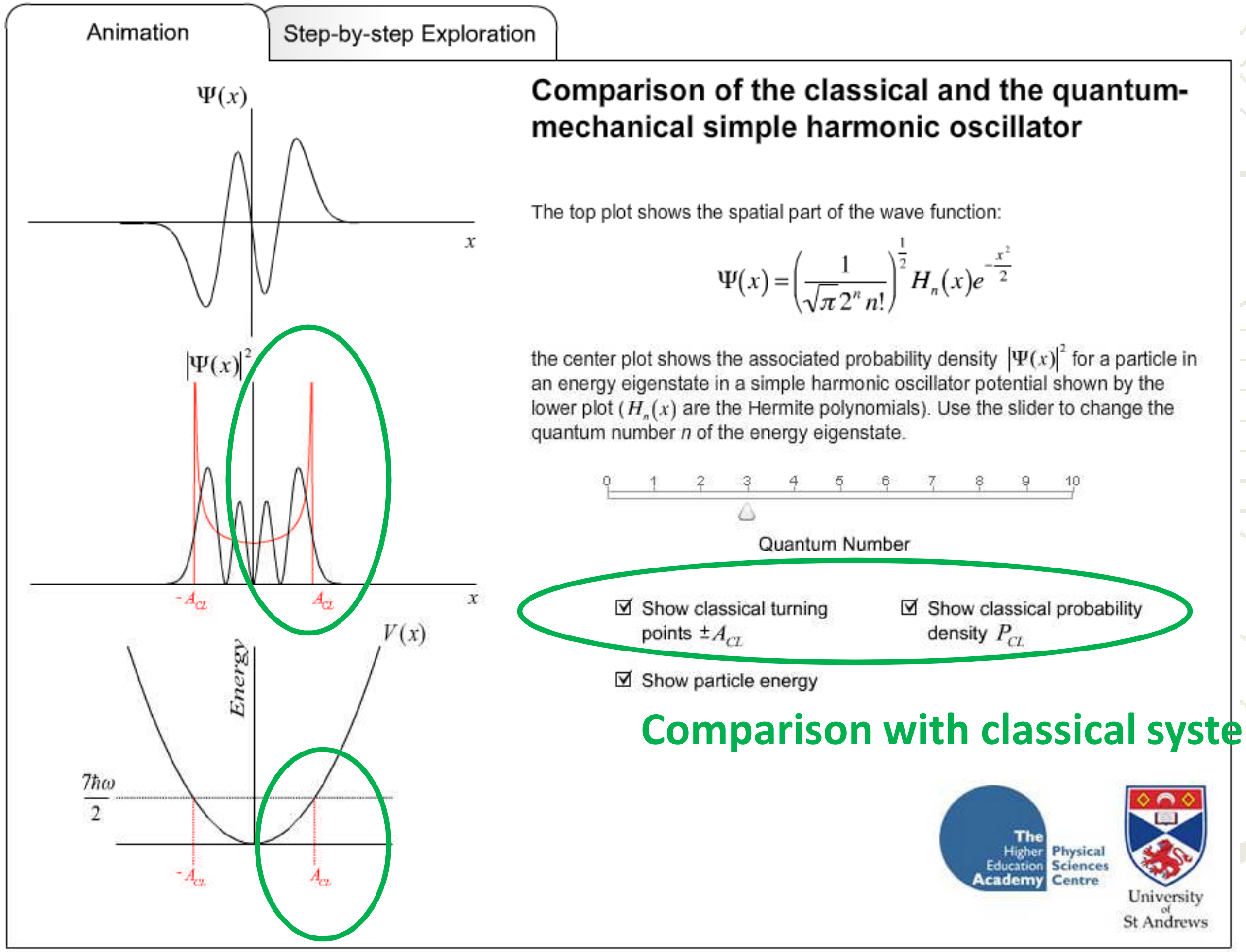
Key features that make the animations effective for learning

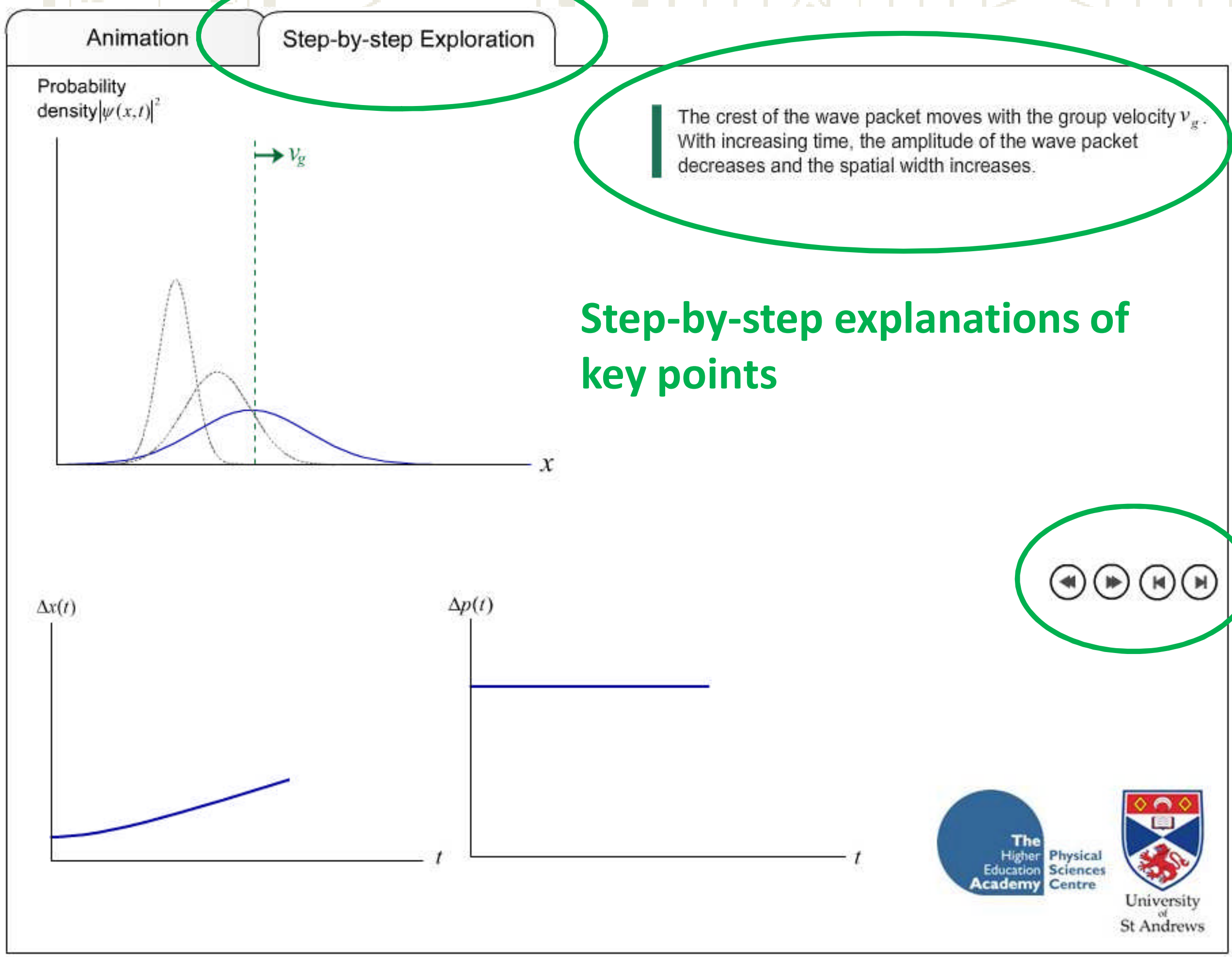


Key features that make the animations effective for learning



Comparison with classical systems

Key features that make the animations effective for learning

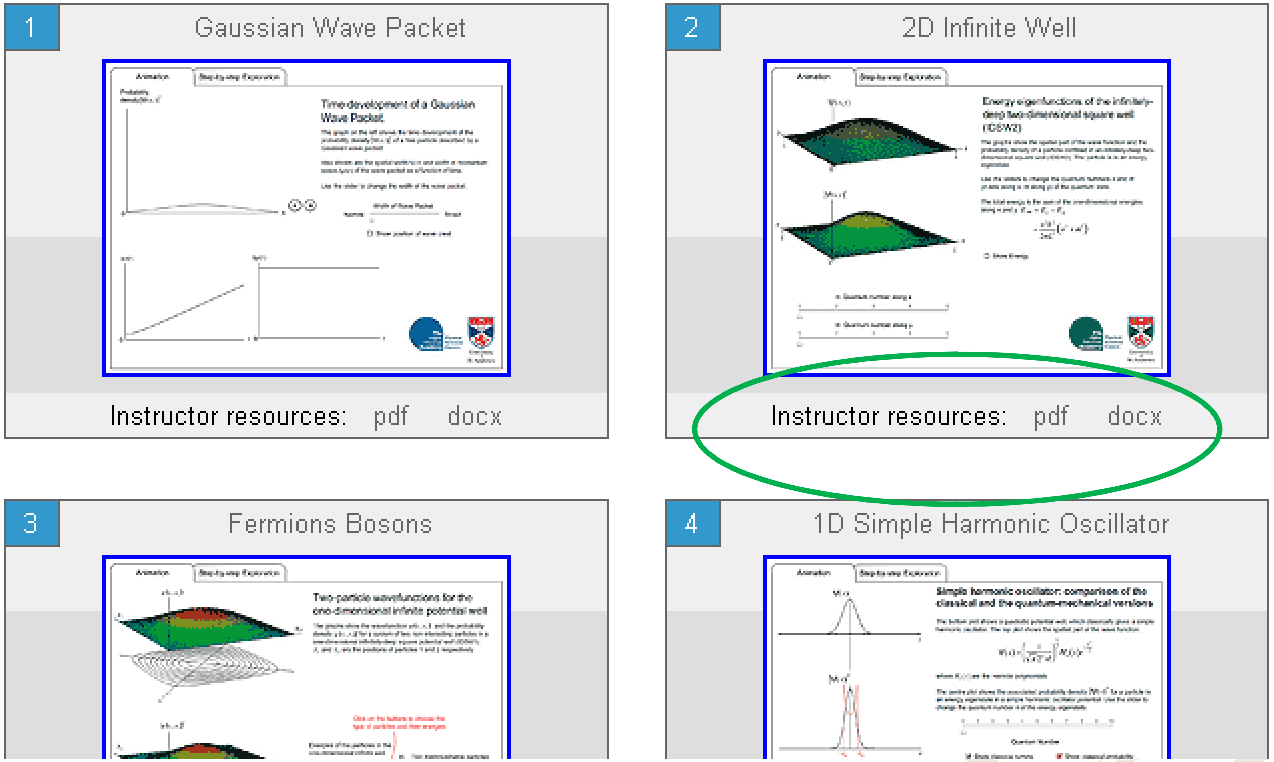


Step-by-step explanations of key points

Free availability of animations and instructor resources

http://www.st-andrews.ac.uk/~qmanim/

Adaptability to a variety of learning goals



Instructor resources (worksheets with full solutions)

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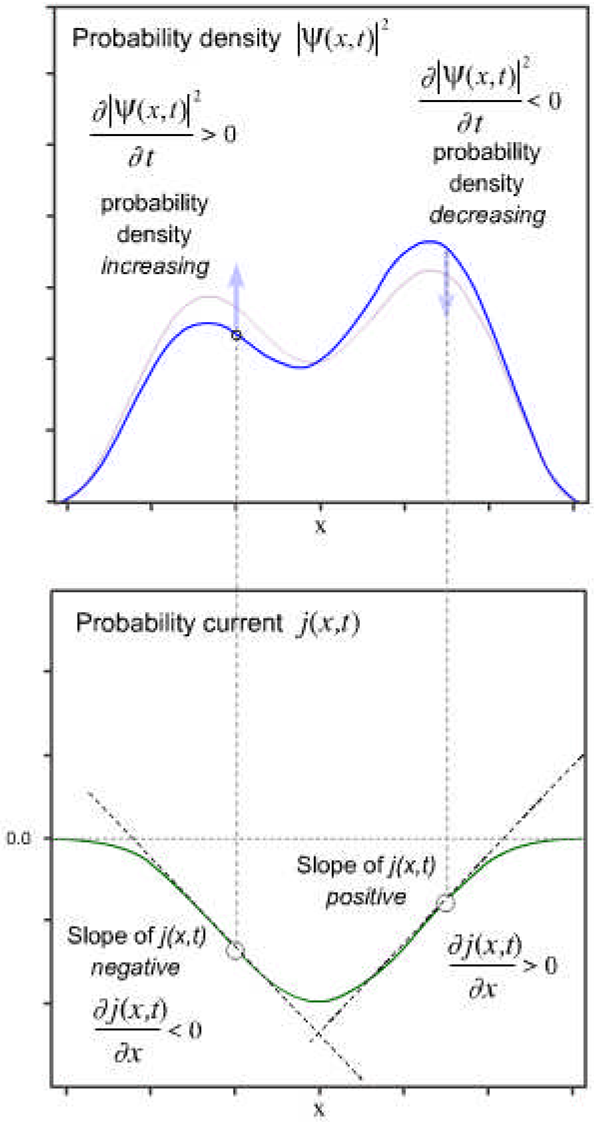
Animation | Step-by-step Exploration



Conceptual relationship between the two plots

This step-by-step exploration shows the relationship between the gradient of the probability current (lower plot) and the rate of change with time of the probability density (upper plot). Use the control buttons below to step forward to the next stage of the explanation.

Slope of the probability current *j(x,t)*:

The slope of *j(x,t)* at the given point is positive.

Change in probability density

The slope of the flux of probability current and the probability density are related by the following expression:

$$\frac{\partial j(x,t)}{\partial x} = -\frac{|\Psi(x,t)|^2}{\partial t}$$

If the slope of *j(x,t)* at a given point is *positive* this implies that the probability density as a function of time at that point is *decreasing*.

Consider a point on the plot of the flux of probability current where the slope of *j(x,t)* is *negative*. We can see that at the corresponding point, the probability density is *increasing* in time.

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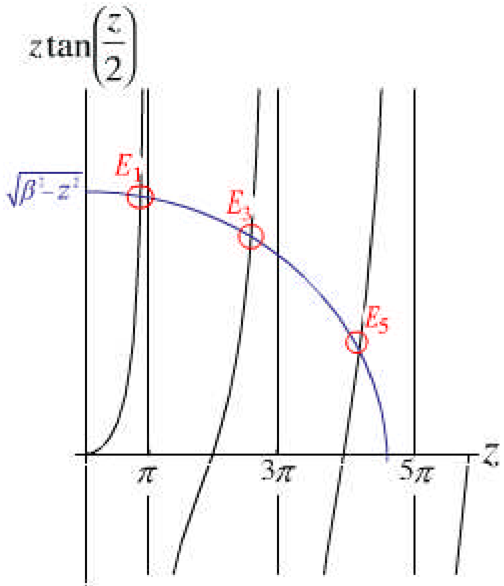
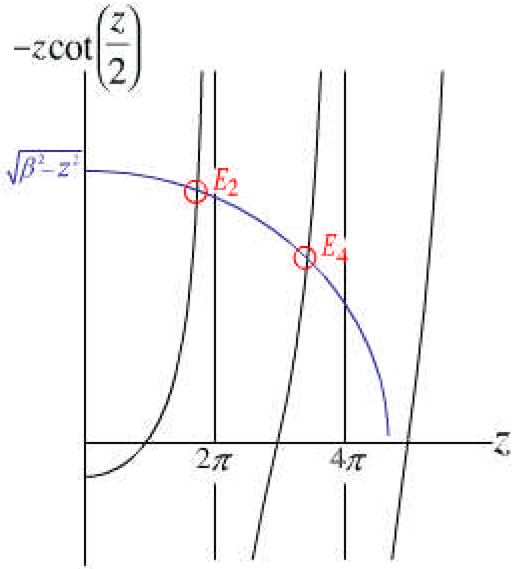
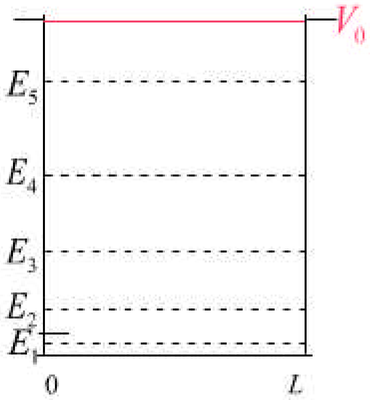
Animation | Step-by-step Exploration







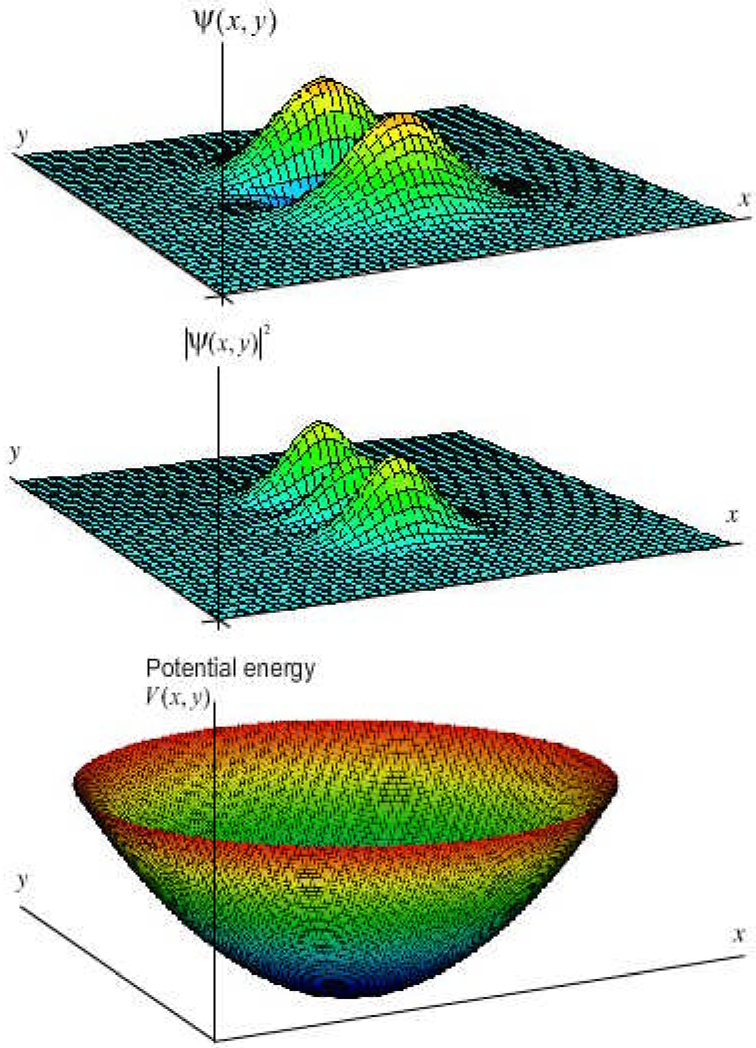
Energy eigenstates in the finite square well.

The figures show a graphical method for determining the energy eigenvalues for a particle of mass m in a one-dimensional finite well of depth V_0. Solutions for bound states in the finite square well need to fulfill either the equation $z\tan\left(\frac{z}{2}\right)=\sqrt{\beta^2-z^2}$ or the equation $-z\cot\left(\frac{z}{2}\right)=\sqrt{\beta^2-z^2}$, where $\beta^2=\frac{2mV_0L^2}{\hbar^2}$ is proportional to the well depth. There is no solution in closed form, but the equations can be solved graphically as the intersection points of the curves. This is shown in the two figures on the left.

Use the slider to change the depth of the well.

$V_0 =$ 1 4 9 16 $\frac{\pi^2\hbar^2}{2mL^2}$

Animation | **Step-by-step Exploration**



The total wave function is the *product* of the one-dimensional wave functions along x and y: $\Psi_{nm}(x,y) = \Psi_n(x)\,\Psi_m(y)$. The quantum numbers *n* and *m* can be varied independently.

The total energy is the sum of the one-dimensional energies along x and y:

$$E_{nm} = E_n + E_m = \hbar\omega\left(n + \frac{1}{2}\right) + \hbar\omega\left(m + \frac{1}{2}\right) = \hbar\omega(n + m + 1)$$

where ω is angular frequency which depends on the strength of the potential and the mass of the particle. Different combinations of the quantum numbers *n* and *m* can have the same energy: such states are said to be *degenerate*.

Check the "Show Energy" box below to show the energy as you change the quantum numbers using the sliders.

$$E_{nm} = E_n + E_m = \hbar\omega(n + m + 1)$$

$$E_{tot} = \hbar\omega(0 + 2 + 1)$$

$$= 3\hbar\omega$$

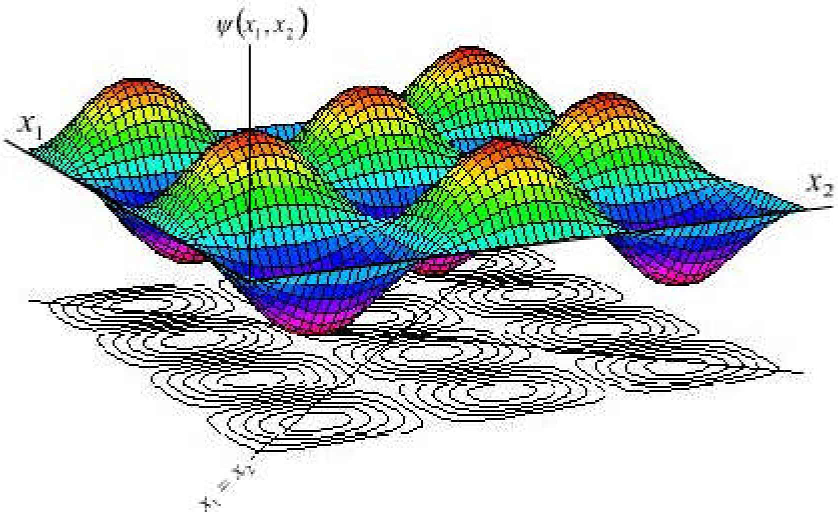
n: Quantum number along x

0 1 2 3 4 5

m: Quantum number along y

0 1 2 3 4 5

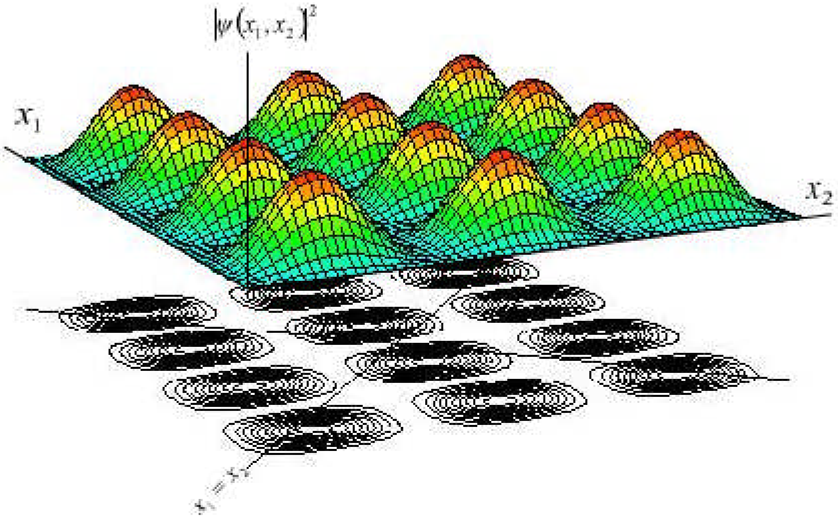
Animation | **Step-by-step Exploration**



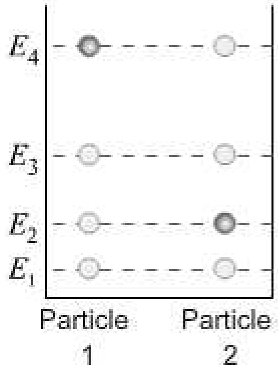
If the particles are indistinguishable, the probability density under exchange of the particles must remain the same. Therefore the wave function under particle exchange must remain unchanged excepting a possible factor of -1.

For fermions (half-integer spin particles), the total wave function is antisymmetric, i.e., $\psi(x_1, x_2) = \frac{1}{\sqrt{2}}\left(\phi_1(x_1)\ \phi_2(x_2) - \phi_1(x_2)\ \phi_2(x_1)\right)$, where ϕ_i is the wave function of the single-particle state *i*. In particular, two fermions cannot be in the same quantum state - the wave function is equal to zero. If the two single-particle states are different, the antisymmetry requirement leads to a suppression of probability for the two particles to be in the same place compared with two distinguishable particles.

This can be seen in the reduction of the probability density along the line $x_1 = x_2$ for fermions compared with distinguishable particles.



Energies of the particles in the one-dimensional infinite well



- (●) Two distinguishable particles
- ( ) Two indistinguishable fermions (with parallel spins)
- ( ) Two indistinguishable spinless bosons

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Topics of the animations

- Bound states in 1D potentials
- 1D scattering
- Time dependent phenomena
- Measurement
- 2D potentials
- Perturbation theory
- Spin and angular momentum
- Multiparticle wavefunctions

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Evaluating educational effectiveness

- Use of animations in lectures and tutorials in a level 3 course, use of two animations (Finite Well, Potential Step) in a workshop in level 2.
- Questionnaires on student use of and attitudes towards the animations
- Diagnostic survey, administered to level 2 (pre- and post-instruction) and level 3 students.

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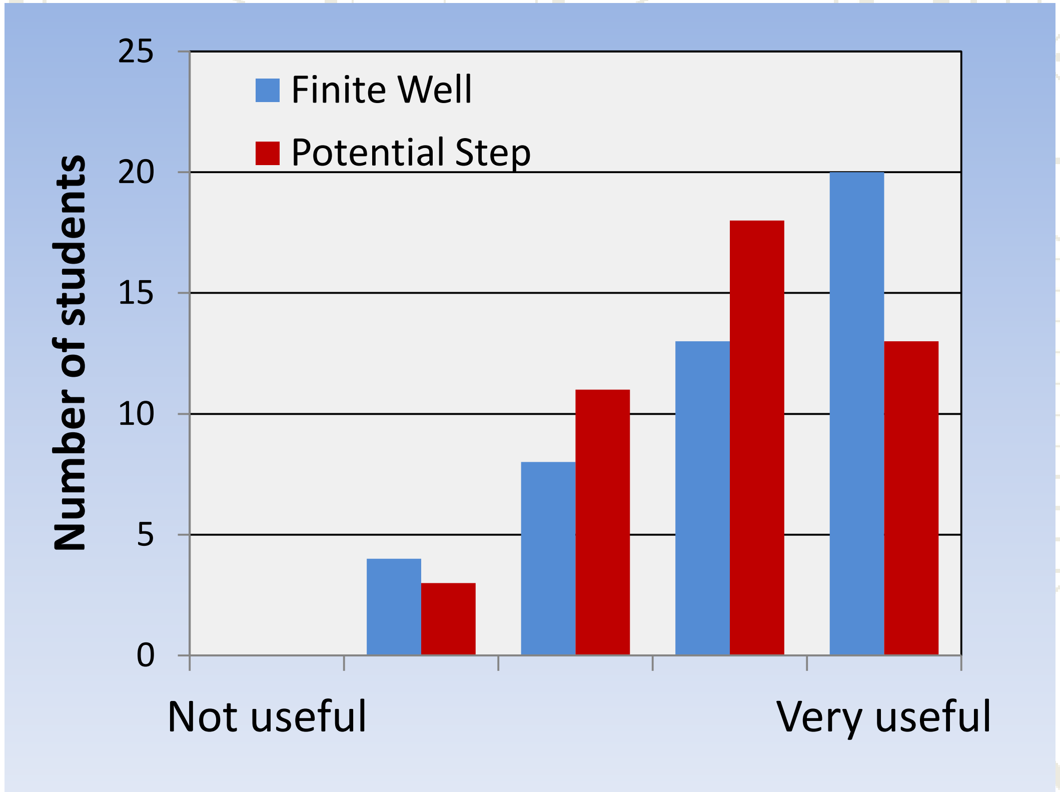
Level 2 questionnaire outcomes



“They were incredibly useful. It's good to get "hands on" with what sometimes feels like a "hands off" topic.”

“I was especially confused in visualizing solutions for the FDSW1 [1D finite-depth square well], but animations of the graphs really helped me understand the concepts“

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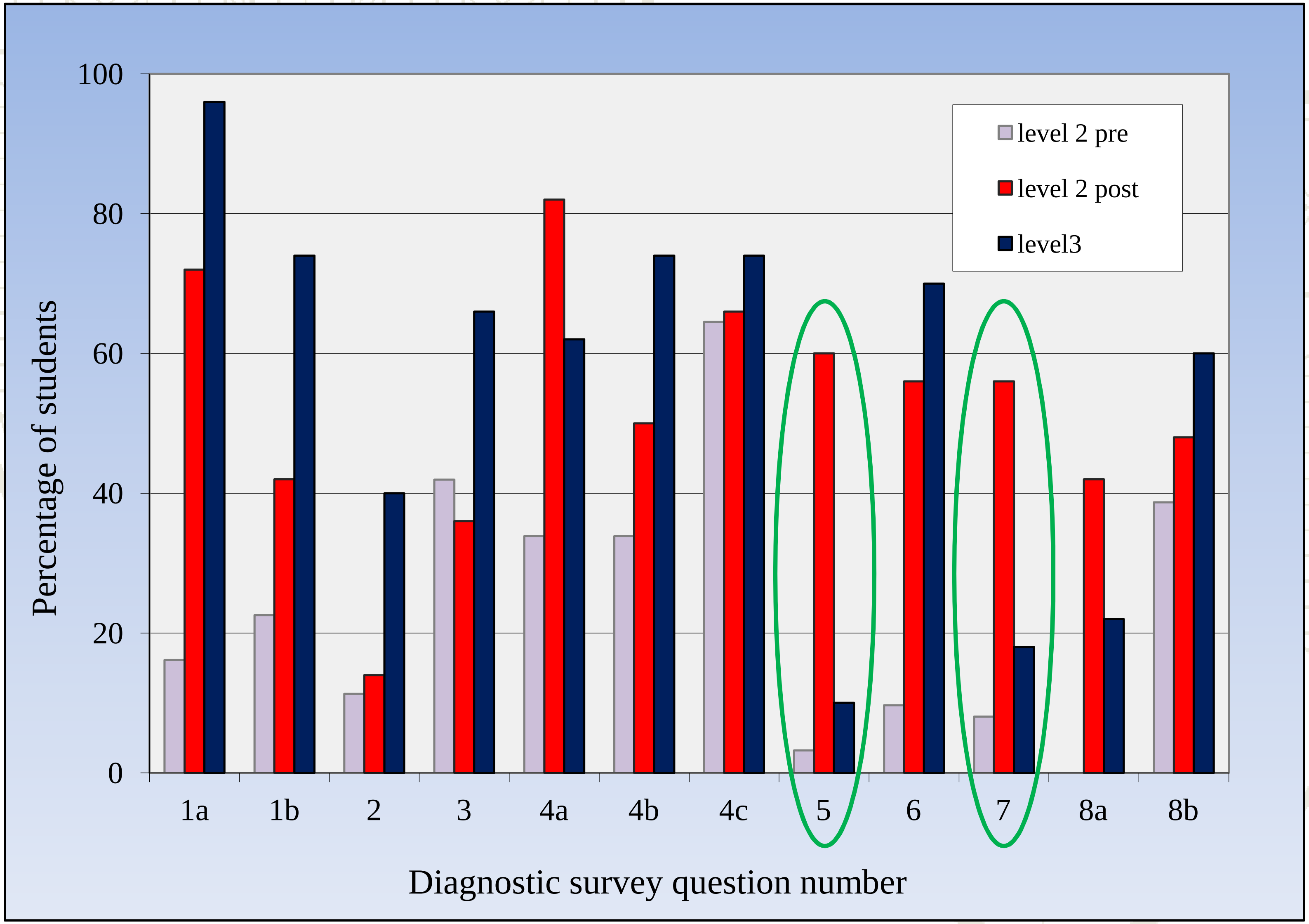
Diagnostic survey outcomes



5: Finite Well **7: Potential Step**

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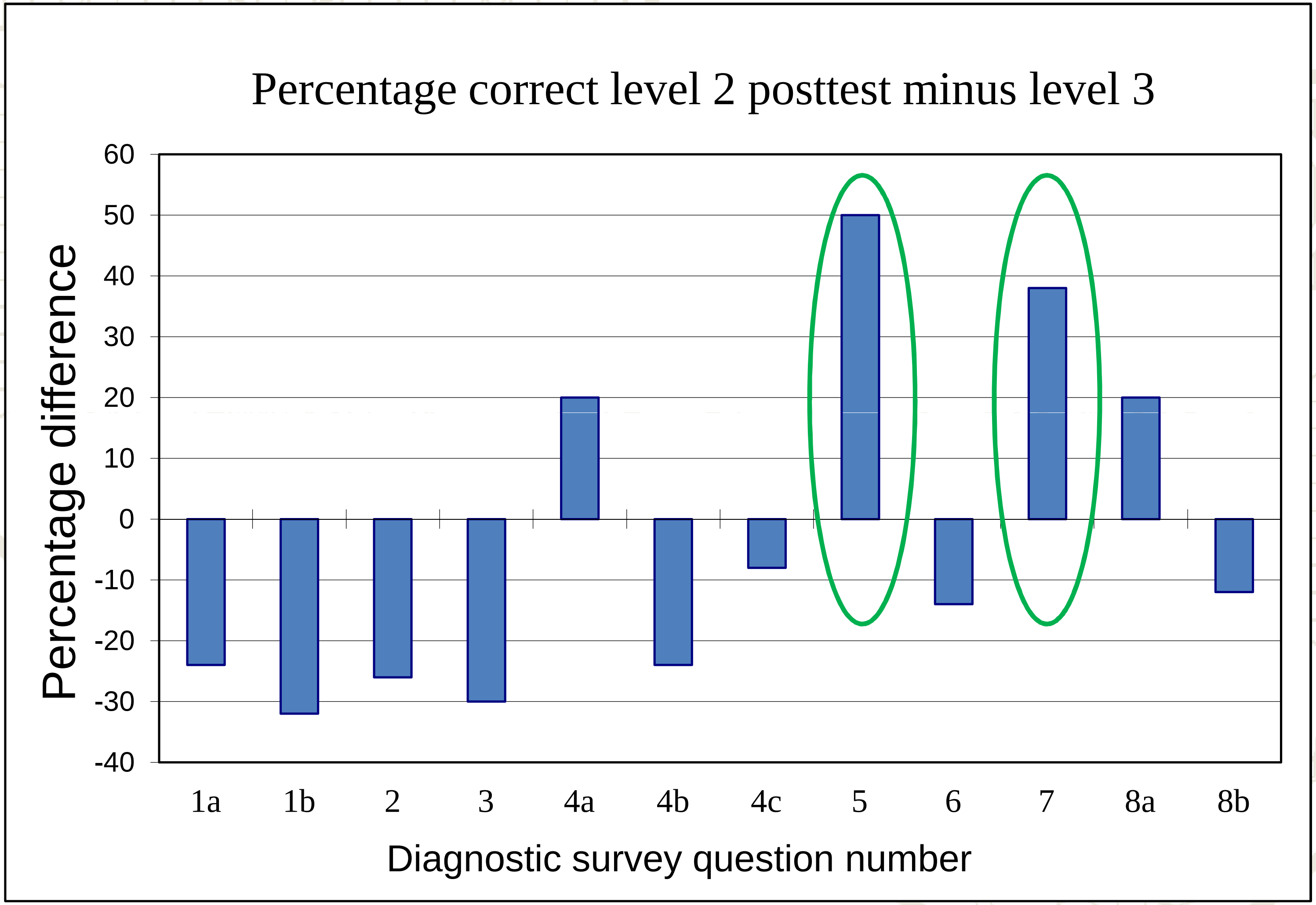
Diagnostic survey outcomes



5: Finite Well **7: Potential Step**

Animations used by level 2, but not by level 3, students

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Future plans

- Extend range of topics and levels covered by the animations (3D scattering, quantum information, classical probability densities, ...)
- Study in depth how students interact with the animations, with the aim to optimizing content and interface.
- Extend animation website functionality.
- Build up a community of users; user input.

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Thanks to

- The Higher Education Academy
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- Emma Robertson, Yuan Deng, Liam Atkinson, Joe Llama (University of St Andrews)

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